Chapter 13

Simple Harmonic Motion

Hooke's Law

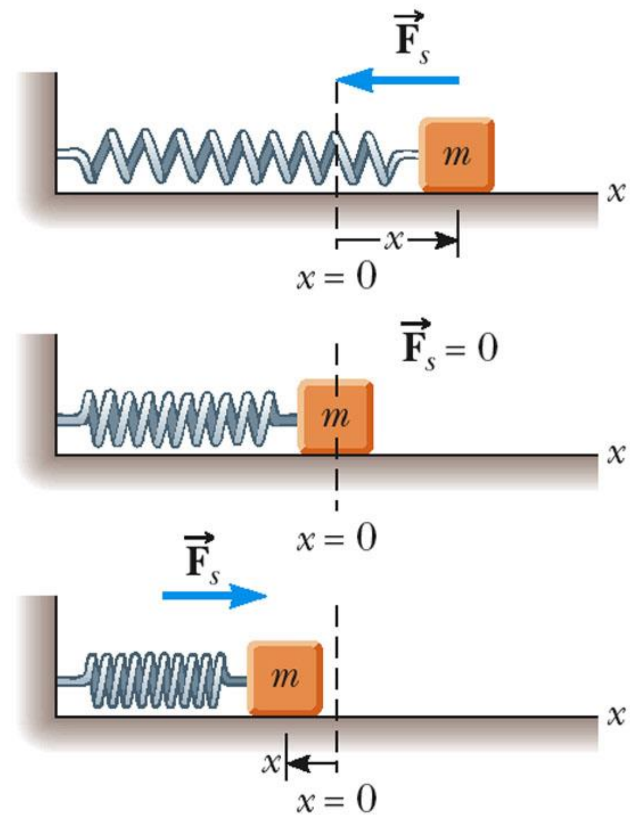
- $F_s = - k\ x$
  - F_s is the spring force
  - k is the spring constant
    - It is a measure of the stiffness of the spring
      - A large k indicates a stiff spring and a small k indicates a soft spring
  - x is the displacement of the object from its equilibrium position
    - x = 0 at the equilibrium position
  - The negative sign indicates that the force is always directed opposite to the displacement

Hooke’s Law Force

- The force always acts toward the equilibrium position
  - It is called the *restoring force*
- The direction of the restoring force is such that the object is being either pushed or pulled toward the equilibrium position

Hooke's Law Applied to a Spring – Mass System

- When x is positive (to the right), F is negative (to the left)
- When x = 0 (at equilibrium), F is 0
- When x is negative (to the left), F is positive (to the right)



Example 1

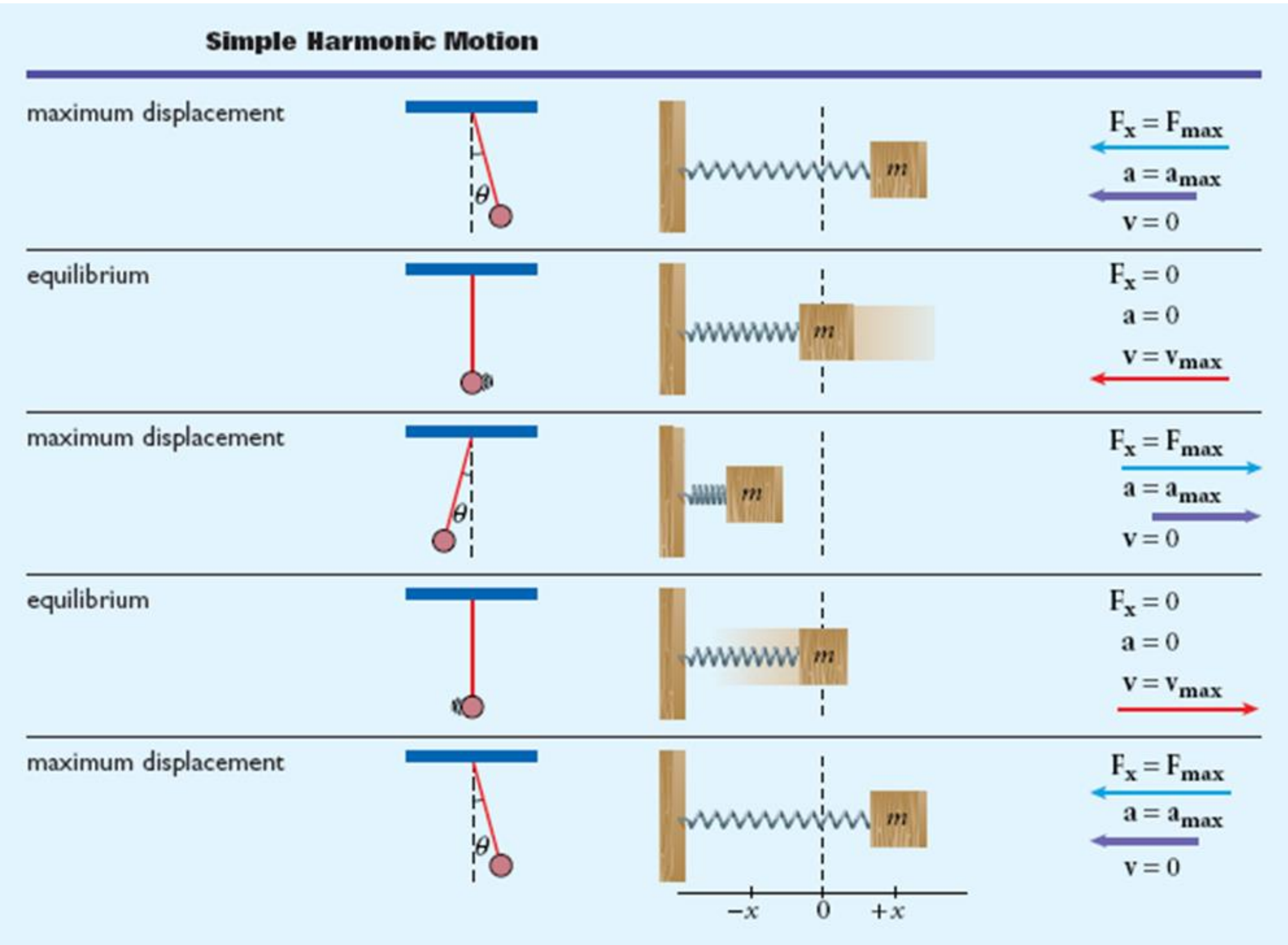
- A 0.5 kg object is attached to a spring with a spring constant 150 N/m so that the object is allowed to move on a horizontal frictionless surface. The object is released from rest when the spring is compressed 0.1 m.
- Find the force on the object.
- What is the acceleration at this instant?

Motion of the Spring-Mass System

- Assume the object is initially pulled to a distance A and released from rest
- As the object moves toward the equilibrium position, F and a decrease, but v increases
- At $x = 0$, F and a are zero, but v is a maximum
- The object's momentum causes it to overshoot the equilibrium position

Motion of the Spring-Mass System, cont

- The force and acceleration start to increase in the opposite direction and velocity decreases
- The motion momentarily comes to a stop at $x = -A$
- It then accelerates back toward the equilibrium position
- The motion continues indefinitely



Simple Harmonic Motion
maximum displacement
θ
m
$F_x = F_{max}$
$a = a_{max}$
$v = 0$
equilibrium
m
$F_x = 0$
$a = 0$
$v = v_{max}$
maximum displacement
θ
m
$F_x = F_{max}$
$a = a_{max}$
$v = 0$
equilibrium
m
$F_x = 0$
$a = 0$
$v = v_{max}$
maximum displacement
θ
m
$F_x = F_{max}$
$a = a_{max}$
$v = 0$
$-x$
0
$+x$

Simple Harmonic Motion

- Motion that occurs when the net force along the direction of motion obeys Hooke's Law
  - The force is proportional to the displacement and always directed toward the equilibrium position
- The motion of a spring mass system is an example of Simple Harmonic Motion

Simple Harmonic Motion, cont.

- Not all periodic motion over the same path can be considered Simple Harmonic motion
- To be Simple Harmonic motion, the force needs to obey Hooke’s Law

Amplitude

- Amplitude, A
  - The amplitude is the maximum position of the object relative to the equilibrium position
  - In the absence of friction, an object in simple harmonic motion will oscillate between the positions $x = \pm A$

Period and Frequency

- The period, T, is the time that it takes for the object to complete one complete cycle of motion
  - From x = A to x = - A and back to x = A
- The frequency, f, is the number of complete cycles or vibrations per unit time
  - $f = 1 / T$
  - Frequency is the reciprocal of the period

Acceleration of an Object in Simple Harmonic Motion

- Newton's second law will relate force and acceleration
- The force is given by Hooke's Law
- $F = - k x = m a$
  - $a = -kx / m$
- The acceleration is a function of position
  - Acceleration is *not* constant and therefore the uniformly accelerated motion equation cannot be applied

Elastic Potential Energy

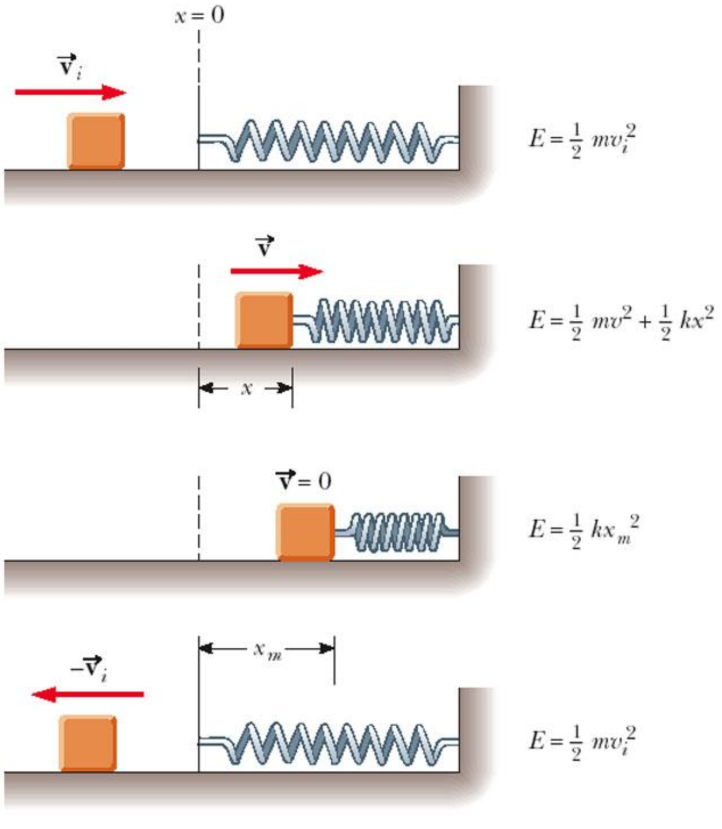
- A compressed spring has potential energy
  - The compressed spring, when allowed to expand, can apply a force to an object
  - The potential energy of the spring can be transformed into kinetic energy of the object

Elastic Potential Energy, cont

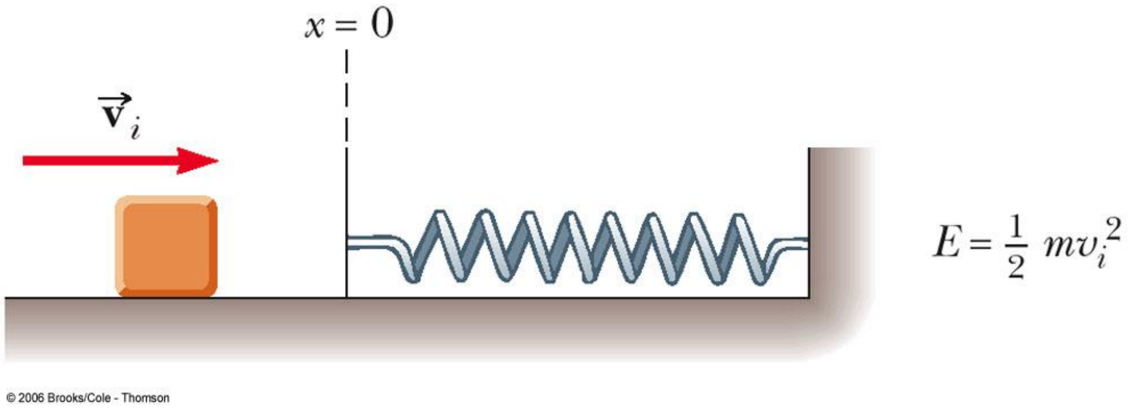
- The energy stored in a stretched or compressed spring or other elastic material is called *elastic potential energy*
  - $PE_s = \frac{1}{2}kx^2$
- The energy is stored only when the spring is stretched or compressed
- Elastic potential energy can be added to the statements of Conservation of Energy and Work-Energy

Energy in a Spring Mass System

- A block sliding on a frictionless system collides with a light spring
- The block attaches to the spring
- The system oscillates in Simple Harmonic Motion

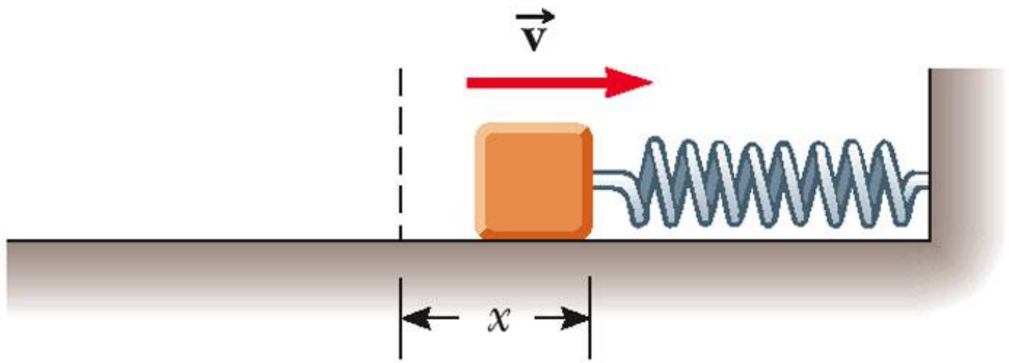


Energy Transformations



- The block is moving on a frictionless surface
- The total mechanical energy of the system is the kinetic energy of the block

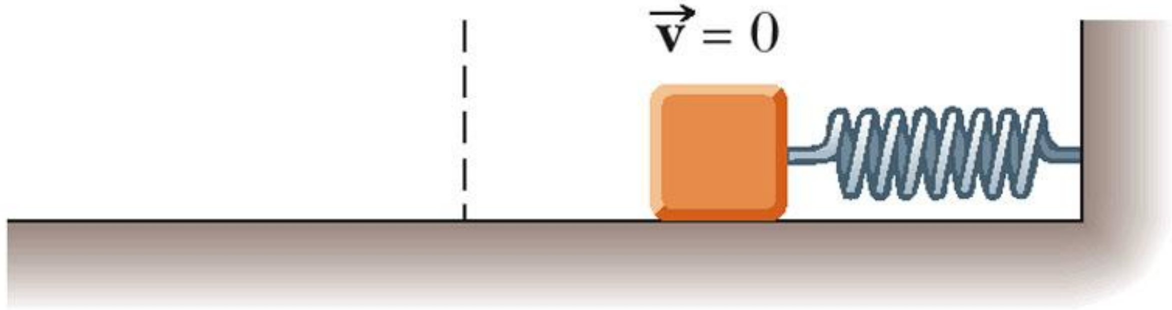
Energy Transformations, 2



$$E = \frac{1}{2} mv^2 + \frac{1}{2} kx^2$$

- The spring is partially compressed
- The energy is shared between kinetic energy and elastic potential energy
- The total mechanical energy is the sum of the kinetic energy and the elastic potential energy

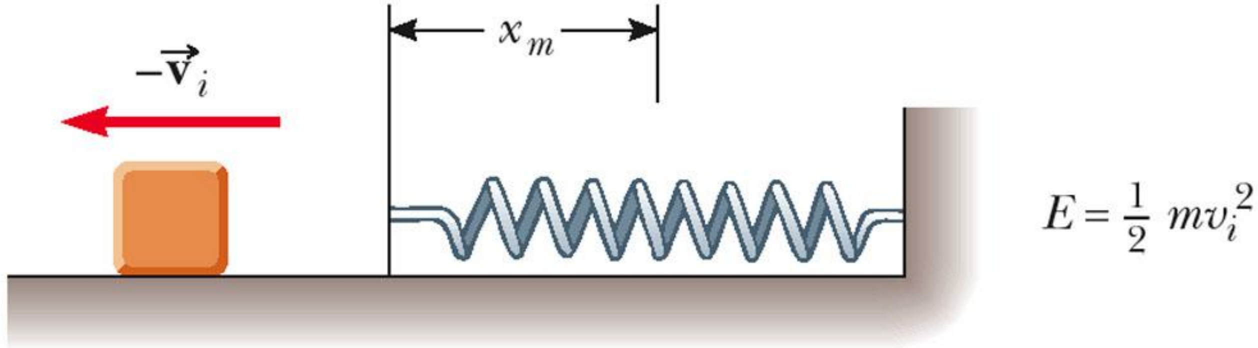
Energy Transformations, 3



$$E = \frac{1}{2} k x_m^{\ 2}$$

- The spring is now fully compressed
- The block momentarily stops
- The total mechanical energy is stored as elastic potential energy of the spring

Energy Transformations, 4



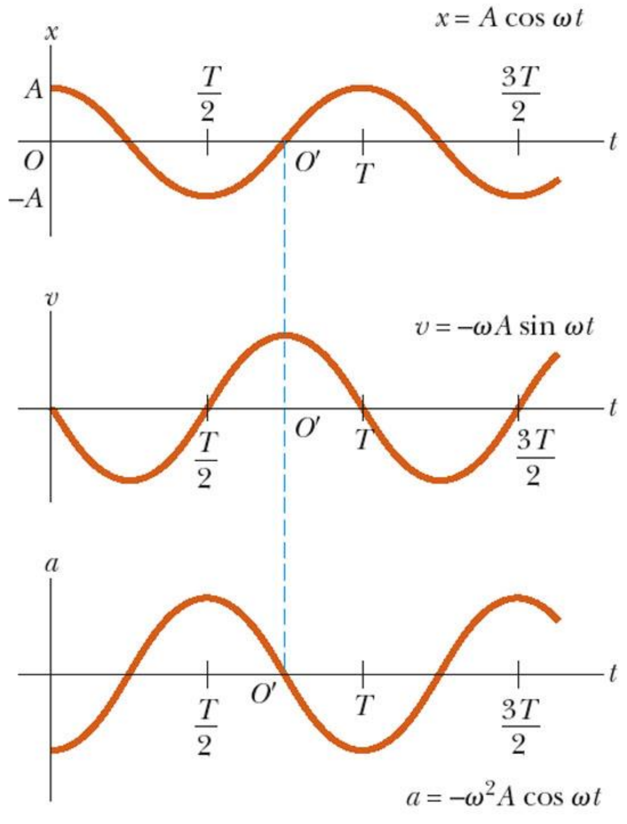
- When the block leaves the spring, the total mechanical energy is in the kinetic energy of the block
- The spring force is conservative and the total energy of the system remains constant

Example 2

- At an outdoor market, a bunch of bananas is set into oscillatory motion with an amplitude of 10 cm on a spring with a spring constant of 20 N/m. It is observed that the maximum speed of the bunch of bananas is 20 cm/s. The acceleration of gravity is 10 m/s^2 .
- What is the weight the bananas?

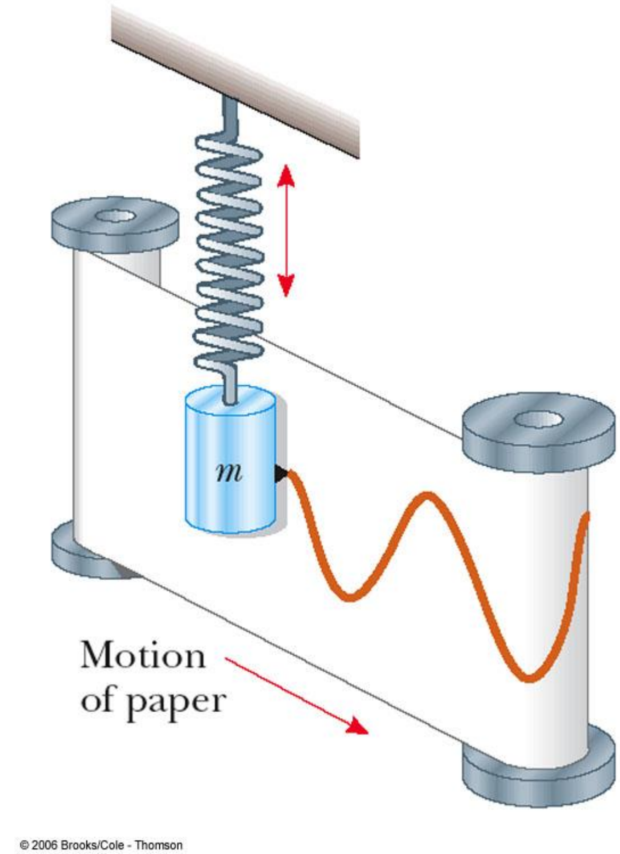
Graphical Representation of Motion

- When x is a maximum or minimum, velocity is zero
- When x is zero, the velocity is a maximum
- When x is a maximum in the positive direction, a is a maximum in the negative direction

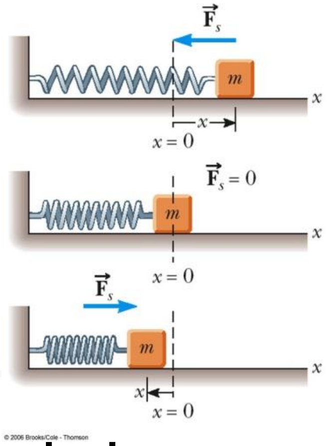


Verification of Sinusoidal Nature

- This experiment shows the sinusoidal nature of simple harmonic motion
- The spring mass system oscillates in simple harmonic motion
- The attached pen traces out the sinusoidal motion

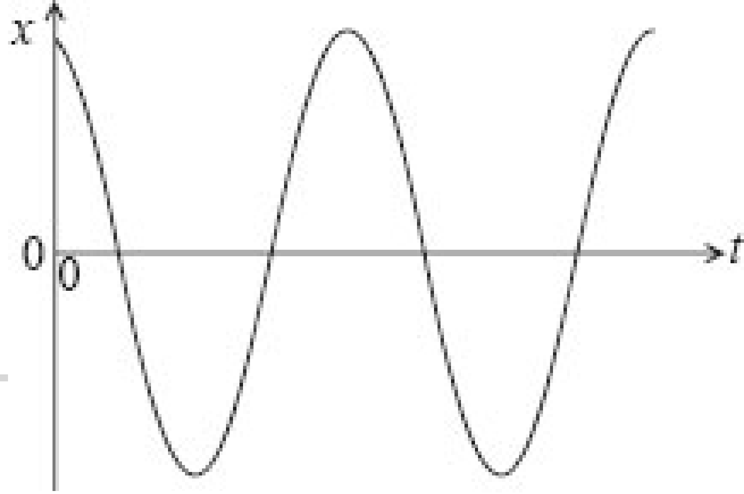


Example 4

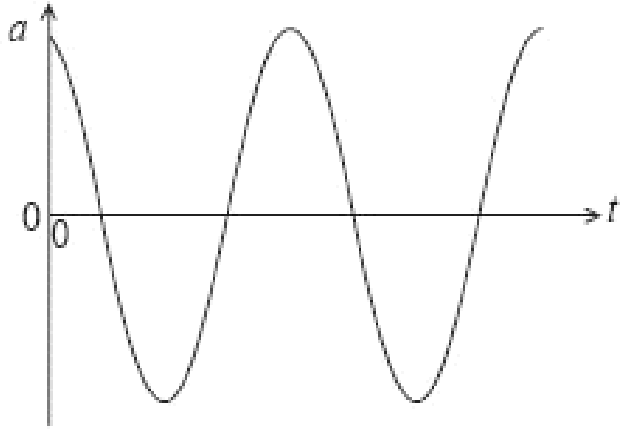


- A 2.0 kg object on a frictionless horizontal track is attached to the end of a horizontal spring whose force constant is 4.5 N/m. The object is displaced 3.0 m to the right from its equilibrium position and then released, which initiates simple harmonic motion.

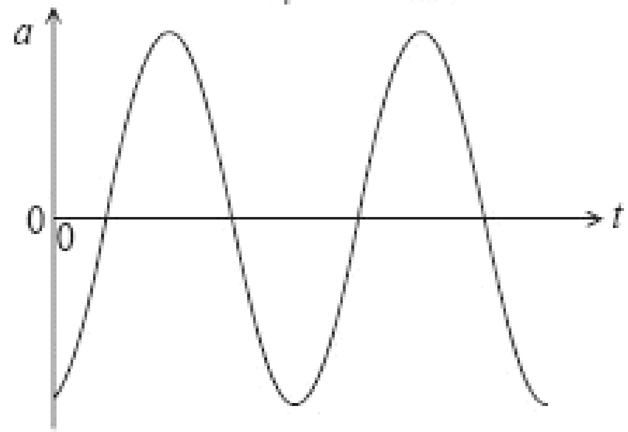
The graph to the right shows the variation with time *t* of the displacement *x* of a particle undergoing simple harmonic motion.



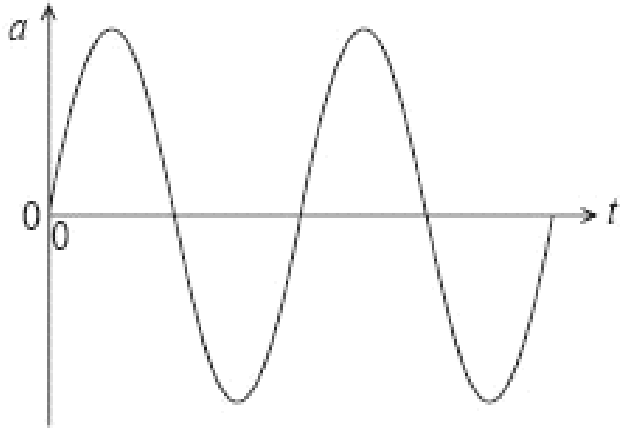
A.



B.



C.



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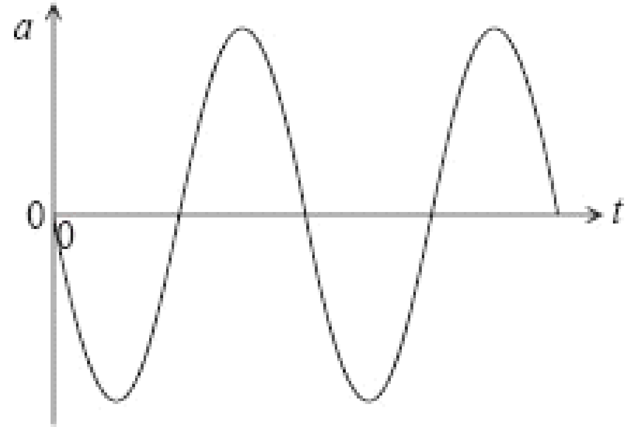

Which graph correctly shows the variation with time *t* of the acceleration *a* of the particle?