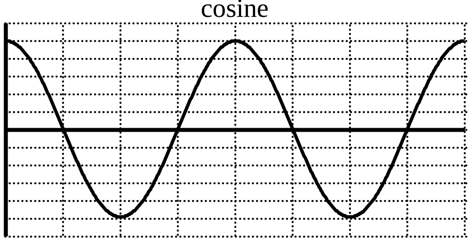
Name________________________________ Date__________ Period_____

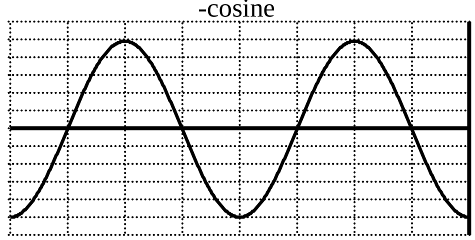
Waves Unit I Activity: Kinematic Equations for SHM

You have seen four different graphs in the work you have done on mass-spring systems oscillating in simple harmonic motion (SHM). Now we will name the four different shapes. The basic trigonometric curves are the cosine and the sine. The negative of each is simply the original flipped over. Below are the shapes of the 4 graphs you will need to recognize.

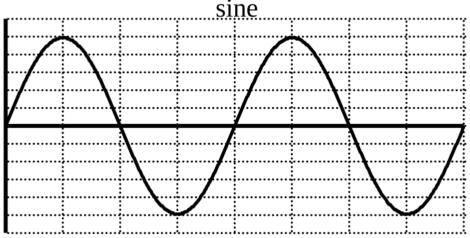
cosine



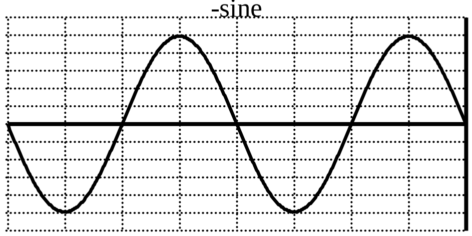
-cosine



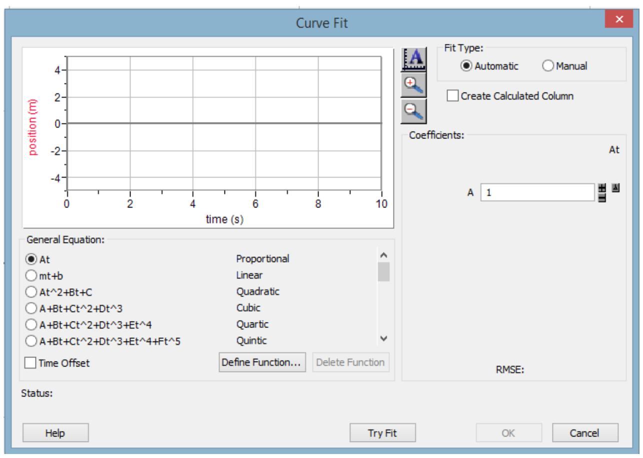
sine

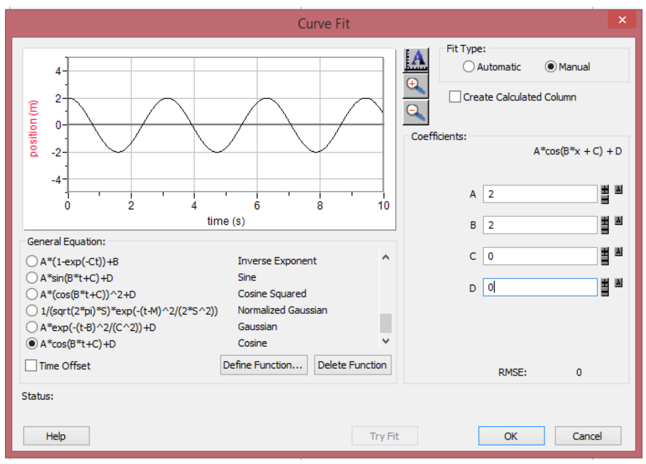


-sine

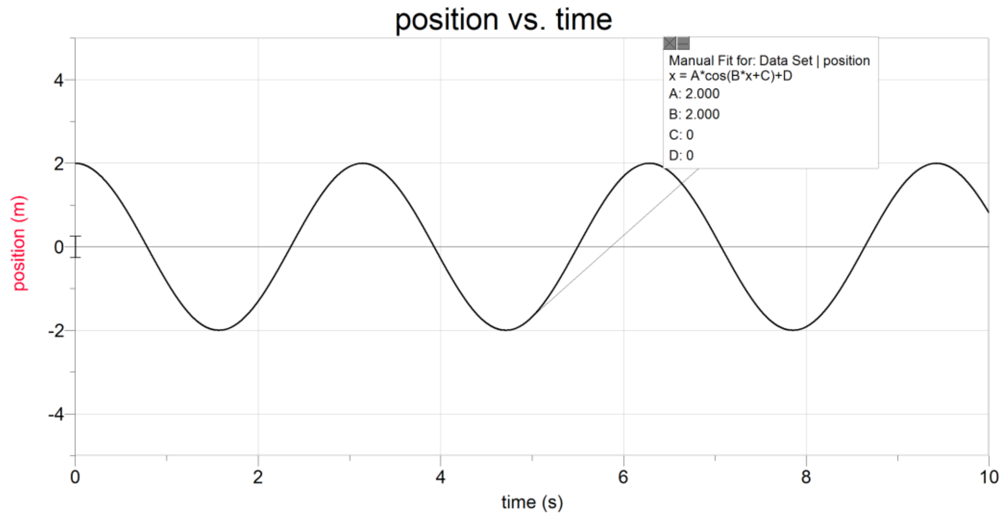


The kinematic equation for position during simple harmonic motion is of the form x = Acos(Bt+C) + D for oscillations that are at maximum amplitude at time t = 0. You will begin by determining what attribute each letter controls. In Logger Pro, open the file "Practice Trig Curves.cmbl". A blank graph titled position vs. time will be loaded. Click on "Analyze" at the top and then "Curve Fit". You should see a window similar to the one below left. Scroll to the bottom of the list and click on Acos(Bt+C) + D as shown in the window below right. In the boxes that show up to the right place a 2 in box A, a 2 in box B, a 0 in boxes C and D. Then click on OK.





The graph at the top of the next page should appear.



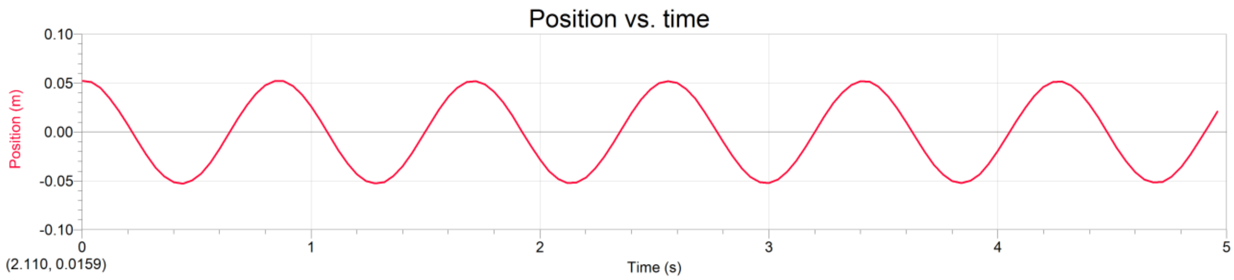
There should be a box with the numbers you entered to make the graph. If you click on a number a small box appears and the number can be changed.

1. Change D to 1 and then to 2. What does D control?
2. Change D back to 0. Change C to 1.57, then 3.14 then 4.71. What does C control? By how much of a cycle did the graph change each time?
3. Change C to -1.57. How was the change of the graph different than when C was 1.57?

For the rest of this activity C and D will remain zeros.

4. C and D should be 0 and B should be 2. Change A to 1, then to 3, and then to 4. What attribute does A control?
5. Change A to -2. How was the change of the graph different than when A was 2?
6. C and D should be 0 and A should be 2. Using numbers of your choice increase and decrease B. What happens to the period and frequency when B is increased?
7. Click on the X in the box titled Manual Fit so the graph disappears. Go to Analyze → Curve Fit again and this time choose Asin(Bt+C) + D. Place 2, 2, 0, 0 in the 4 boxes as before. How is the graph different than before?
8. Change A to -2? How did the graph change from what you had in 7?

You should now know what changing B does to the frequency and the period. We will now look at some graphs you have seen in this unit and analyze them and investigate what B represents. In Logger Pro open the file "Oscillation kinematics graphs.cmbl". In it are graphs of position, velocity and acceleration graphs vs. time produced by a 0.600kg mass oscillating on a spring with a constant of 32.5N/m. Below is the top graph you should see.



Click on the position-time graph, click on Analyze→ then on Curve Fit. Since the graph is a cosine curve choose Acos(Bt+C) + D. Place a 0 in C and D.

9. Do a curve fit for the graph. When you are successful a black graph will overlap the red graph in the curve fit window. Begin by changing A and then change B. When the fit looks good, click on OK to go back to the main window. If the fit can be improved, adjust the numbers in the box until you are satisfied. What is the equation of your graph?

10. What are the units for B? (Hint: Bt can't have units!) If A and B don't have unit in your equation, go back and add them.

11. We still do not know what B represents so let's figure it out. What are $\frac{1}{s}$ the units of?

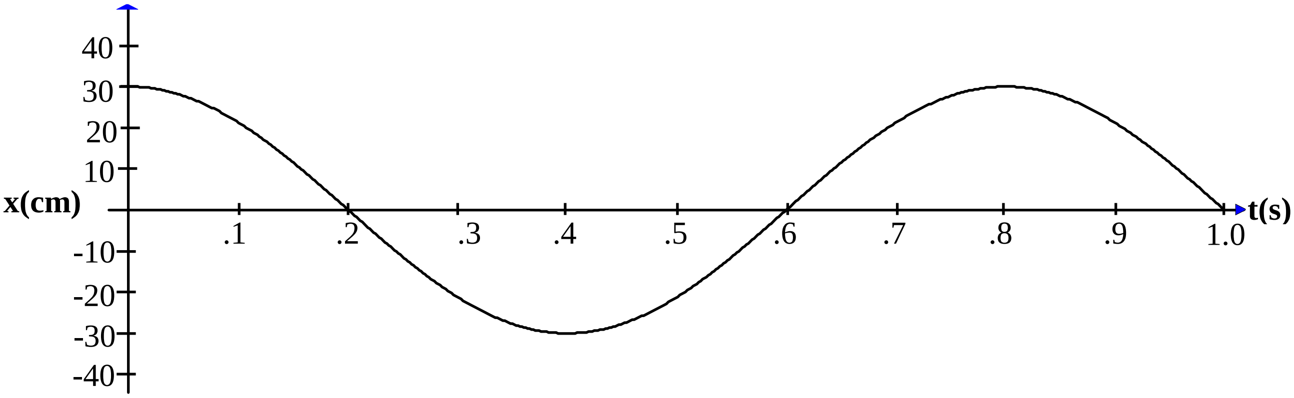
12. Use the examine feature in Logger Pro on the graph to obtain the values you need to solve for the frequency of the oscillations. What do you get for the frequency?

13. Unfortunately the frequency you arrived at does not match the frequency found for B. Divide your value for B by the frequency and see what multiple of the frequency your answer to 12 is.

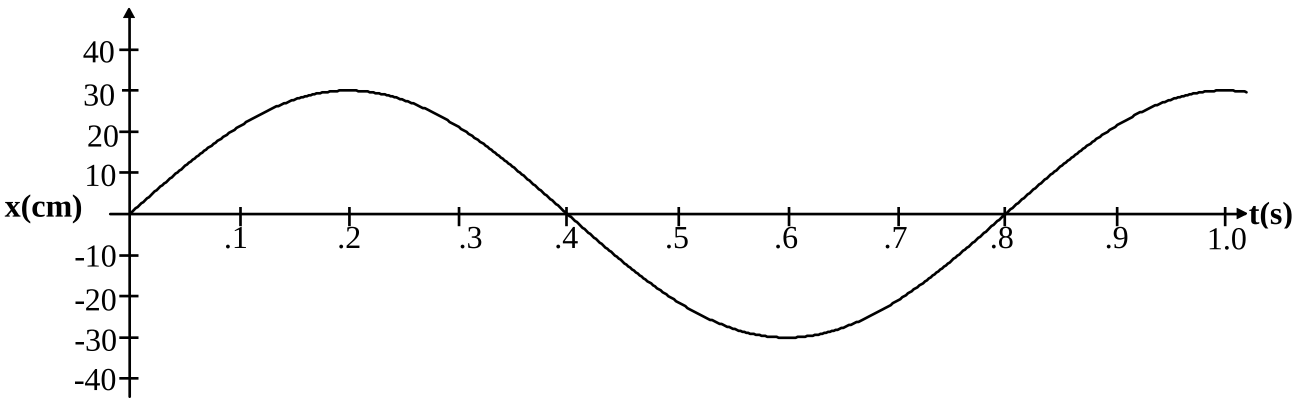
14. Hopefully things have gone well and you recognize that the number you solved for in question 13 is very close to a whole number multiple of pi. What is your equation for B with pi (π) in the equation?

When you get to this point, check with your teacher to make sure you have the correct equation for B. The quantity you have found ($2\pi f$) is known as the angular frequency and is given the symbol omega: ω. The unit of angular frequency is usually said as radians per second and is written as $\frac{1}{s}$.

Now to replace the letters in x = Acos(Bt+C) + D with the quantities they represent. You will only be expected to write equations for graphs where C and D are zero. As you have found earlier A is the amplitude and we use an upper case A for the symbol (what a coincidence). B can be replaced with ω or $2\pi f$. We now have the following equations for the position $x = A\cos(2\pi ft)$ or $x = A\cos(\omega t)$ whenever the mass is at the maximum amplitude at time t = 0. If the graph begins at a different location, the trig function becomes –cos, sin, or –sin.



15. Write the trigonometric equation for position using the graph above:



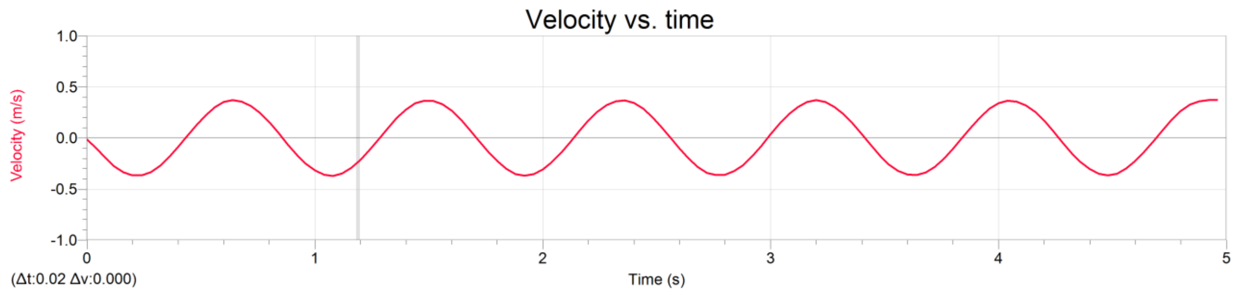
16. Now write the equation for the oscillation shown on this graph:

In question 12 you used time on a graph to get the frequency. Now lets see how you would get the frequency and the angular frequency using the mass and spring constant.

17. The equation $T = 2\pi\sqrt{\frac{m}{k}}$ is one you have used many times in this unit. Now take that equation and use it to solve for the frequency.

18. Now take that equation and take it times 2π to get an equation for ω, the angular frequency.

19. Using the spring constant of $32.5\frac{\text{N}}{\text{m}}$, the oscillating mass of 0.600kg and the equation in 18, solve for the angular frequency. How does it agree with the value in the curve fit?



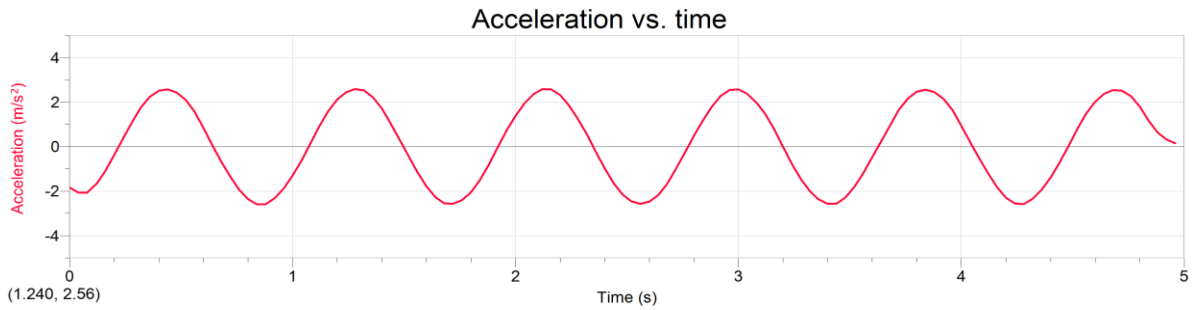
You will now analyze the velocity-time graph. Click on the velocity-time graph. Since this is a –sine graph for your curve fit you should choose General Equation Asin(Bt+C) + D. C and D are 0, so you need to get the correct values for A and B.

20. When the fit looks good, click on OK to go back to the main window. If the fit can be improved, adjust the numbers in the box until you are satisfied. What is the equation of your graph?

21. What does the amplitude of your velocity graph represent?

22. Using an energy approach and the amplitude of the oscillation solve for the maximum speed of the mass during the oscillation. Does it agree with the value for the amplitude of the velocity graph?

23. Your solution to 22 should have involved the equation $v = \sqrt{\frac{k}{m}}A$. Look at 18 to see what $\sqrt{\frac{k}{m}}$ can be replaced with. Using this you should see that the equation can be written as $v_{\text{max}} = \omega A = 2\pi fA$. The general equation for the velocity as a function of time can be written as $v = -v_{\text{max}}\sin(\omega t)$ or $v = -\omega A\sin(\omega t)$ if the oscillation began at the greatest positive position. Why does the equation need the – sign?



You will now analyze the acceleration-time graph. Click on the acceleration-time graph. Since this is a –cosine graph for your curve fit you should choose General Equation Acos(Bt+C) + D. C and D are 0, so you need to get the correct values for A and B.

24. When the fit looks good, click on OK to go back to the main window. If the fit can be improved, adjust the numbers in the box until you are satisfied. What is the equation of your graph?

25. What does the amplitude of your acceleration graph represent?

26. Using a force approach and the amplitude of the oscillation solve for the maximum acceleration of the mass during the oscillation. Does it agree with the value for the amplitude for the acceleration graph?

27. Your solution to 26 should have involved the equation $kA = ma \Rightarrow a = \frac{k}{m}A$. Since $\omega = \sqrt{\frac{k}{m}}$ what can we replace $\frac{k}{m}$ with?

28. You should see that $\frac{k}{m}$ is an equation for omega squared (ω^2), therefore the maximum acceleration can also be solved for using $a_{max} = \omega^2 A$. The general equation for the acceleration as a function of time can be written as $a = -a_{max}\cos(\omega t)$ or $a = -\omega^2 A\cos(\omega t)$ if the oscillation began at the greatest positive position. Why does the equation need the – sign?