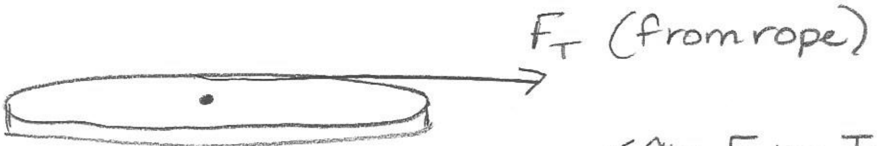
Worksheet 6 – AP Rotational Motion

1.



$\Sigma \tau = F_T r = I\alpha$

$F_T = \frac{I\alpha}{r}$

$F_T = \frac{(169\,kgm^2)(\frac{\pi}{2}\frac{rad}{s^2})}{1.50\,m}$

$\boxed{F_T = 177\,N}$

$I = \frac{1}{2}mr^2$

$I = \frac{1}{2}(150\,kg)(1.50\,m)^2$

$I = 337.5\,kgm^2(\frac{1}{2})$

$I = 338\,kgm^2(\frac{1}{2})$

$I = 169\,kgm^2$

$M = 150\,kg$

$\omega_i = 0\,\frac{rad}{s}$

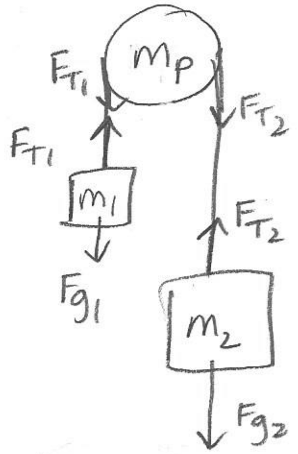
$\omega_f = \frac{0.500\,rev}{s}\left|\frac{2\pi\,rad}{1\,rev}\right. = \frac{\pi\,rad}{s}$

$r = 1.50\,m$

$\Delta t = 2.00\,s$

$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{\pi\frac{rad}{s} - 0\frac{rad}{s}}{2.00\,s} = \frac{\pi}{2}\frac{rad}{s^2}$

2.



$m_1 = 10.0\,kg$

$m_2 = 20.0\,kg$

$m_p = 8.00\,kg$

$r = 0.200\,m$

$a = \alpha r$

a) otherwise rotational will not happen; unbalanced torque is required for rotation.

b) Solve using forces/torques and energy.

Forces/Torques BELOW

$\Sigma \tau = F_{T_2}r - F_{T_1}r = I\alpha$

$F_{T_2}\cancel{r} - F_{T_1}\cancel{r} = \frac{1}{2}m_p r^{\cancel{2}}\left(\frac{a}{\cancel{r}}\right)$

$F_{T_2} - F_{T_1} = \frac{1}{2}m_p a$

$\Sigma F_1 = F_{T_1} - F_{g_1} = m_1 a$ $\quad \Sigma F_2 = F_{g_2} - F_{T_2} = m_2 a$

$F_{T_1} = m_1 a + m_1 g$ $\quad F_{T_2} = m_2 g - m_2 a$

$m_2 g - m_2 a - m_1 a - m_1 g = \frac{1}{2}m_p a$

$g(m_2 - m_1) = a(\frac{1}{2}m_p + m_2 + m_1)$

$10\,N/kg\,(10\,kg) = a(4.00\,kg + 20.0\,kg + 10.0\,kg)$

$100\,N = a(34.0\,kg)$

$\boxed{a = 2.94\frac{m}{s^2}}$

$E_{Before} = E_{after}$

$v_f^2 = \cancel{v_i^2} + 2a\Delta y$ (0)

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$2\alpha\theta = 2\left(\frac{a}{r}\right)\left(\frac{\Delta y}{r}\right)$

$m_2 g\Delta y = \frac{1}{2}m_1 v_f^2 + \frac{1}{2}m_2 v_f^2 + \frac{1}{2}I\omega_f^2 + m_1 g\Delta y$

$m_2 g\Delta y = \frac{1}{2}m_1 v_f^2 + \frac{1}{2}m_2 v_f^2 + \frac{1}{2}\left(\frac{1}{2}m_p \cancel{r^2}\right)\left(\frac{v^2}{\cancel{r^2}}\right) + m_1 g\Delta y$

$m_2 g\cancel{\Delta y} = \cancel{\frac{1}{2}}m_1(\cancel{2}a\cancel{\Delta y}) + \cancel{\frac{1}{2}}m_2(\cancel{2}a\cancel{\Delta y}) + \frac{1}{4}m_p(2a\cancel{\Delta y}) + m_1 g\cancel{\Delta y}$

$g(m_2 - m_1) = a\left(m_1 + m_2 + \frac{1}{2}m_p\right)$

$(10\,N/kg)(10\,kg) = a(34\,kg)$

$$\boxed{a = 2.94\frac{m}{s^2}}$$

c) Use force diagrams/equations

$F_{T_2} = m_2 g - m_2 a = m_2(g - a)$

$F_{T_2} = 20\,kg(10\,N/kg - 2.94\,m/s^2)$

$$\boxed{F_{T_2} = 141\,N}$$

$F_{T_1} = m_1(a + g)$

$F_{T_1} = 10\,kg(10\,N/kg + 2.94\,m/s^2)$

$$\boxed{F_{T_1} = 129\,N}$$

3. 

$KE = \frac{1}{2}I\omega_f^2$

$Fg_M = 800\,N$

$r = 1.50\,m$

$\omega_i = 0\,\frac{rad}{s}$

$F_\perp = 50.0\,N$

$I = \frac{1}{2}mr^2$

$\Delta t = 3.00\,s$

$\tau = I\alpha$

$m = \frac{F}{g} = \frac{800\,N}{10\,N/kg} = 80\,kg$

$KE = \frac{1}{2}I\omega_f^2$

$KE = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(25\,\frac{rad}{s}\right)^2$

$KE = \frac{1}{2}\left(\frac{1}{2}(80\,kg)(1.50\,m)^2\right)\left(25\,\frac{rad}{s}\right)^2$

$$\boxed{KE = 1125\,J = 1130\,J}$$

$\tau = I\alpha$

$Fr = \left(\frac{1}{2}mr^2\right)(\alpha)$

$\alpha = \frac{Fr}{\frac{1}{2}mr^2} = \frac{(50.0\,N)(1.50\,m)}{\frac{1}{2}(80\,kg)(1.50\,m)^2}$

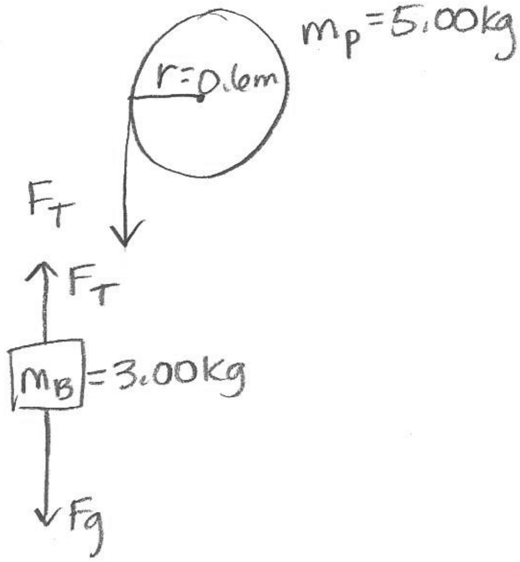
$\alpha = 0.83\,\frac{rad}{s^2}$

$\omega_f = \cancel{\omega_i} + \alpha\Delta t$ (0)

$\omega_f = \left(0.83\,\frac{rad}{s^2}\right)(3.00\,s)$

$\omega_f = 25\,\frac{rad}{s}$

4.



$V_1 = 0\,m/s$

$I = \frac{1}{2} m_p r^2$

$\omega = \frac{V}{r}$

$\Delta y = 4.00m$

$$m_B g \Delta y = \frac{1}{2} m_B V_f^2 + \frac{1}{2} I \omega_f^2$$

$$m_B g \Delta y = \frac{1}{2} m_B V_f^2 + \frac{1}{2}\left(\frac{1}{2} m_p \cancel{r^2}\right)\left(\frac{V_f^2}{\cancel{r^2}}\right)$$

$$m_B g \Delta y = \frac{1}{2} m_B V_f^2 + \frac{1}{4} m_p V_f^2$$

$$m_B g \Delta y = V_f^2 \left(\frac{1}{2} m_B + \frac{1}{4} m_p\right)$$

$$(3kg)(10\,N/kg)(4m) = V_f^2 (1.50kg + 1.25kg)$$

$$V_f = 6.6\,\frac{m}{s}$$

$$\omega_f = \frac{V_f}{r} = \frac{6.6\frac{m}{s}}{0.6m} = \boxed{11\,\frac{rad}{s}}$$

5. $r = 0.180m$

$m = 2.40kg$

$\omega = 35.0\,rad/s$

$L = I\omega$

a) $I = mr^2$

$L = I\omega = mr^2\omega = (2.40kg)(0.180m)^2(35.0\,rad/s)$

$\boxed{L = 2.72\,\frac{kgm^2}{s}}$

b) $I = \frac{1}{2}mr^2 \quad L = I\omega$

$L = \frac{1}{2}mr^2(\omega) = \frac{1}{2}\left(2.72\,\frac{kgm^2}{s}\right) = \boxed{1.36\,\frac{kgm^2}{s}}$

c) $I = \frac{2}{5}mr^2 \quad L = I\omega$

$L = \frac{2}{5}mr^2\omega = \frac{2}{5}\left(2.72\,\frac{kgm^2}{s}\right) = \boxed{1.09\,\frac{kgm^2}{s}}$

d) $I = \frac{2}{3}mr^2 \quad L = I\omega$

$L = \frac{2}{3}mr^2\omega = \frac{2}{3}\left(2.72\,\frac{kgm^2}{s}\right) = \boxed{1.81\,\frac{kgm^2}{s}}$

6. $v = \omega r$

$l = 1.00m$

$m_1 = 4.00kg \quad r_1 = 0.5m$

$m_2 = 3.00kg \quad r_2 = 0.5m$

$v = 5.00 m/s$

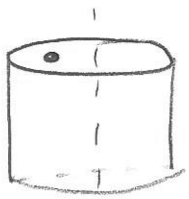
$L = I\omega = (m_1r_1^2 + m_2r_2^2)\,\omega$

$L = (m_1r_1^2 + m_2r_2^2)\left(\frac{v}{r}\right)$

$L = \left((4kg)(0.5m)^2 + (3kg)(0.5m)^2\right)\left(\frac{5.00\frac{m}{s}}{0.5m}\right)$

$\boxed{L = 17.5\frac{kgm^2}{s}}$

7.



$m_c = 10.0kg$

$r_c = 1.00m$

$\omega_c = 7.00\frac{rad}{s}$

$I_c = \frac{1}{2}mr^2$

$m_p = 0.250kg$

$r = 0.900m$

ω_f?

$I_p = mr^2$

$L_i = L_f$

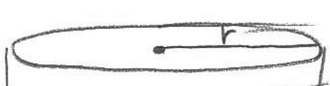
$I_i\omega_i = I_f\omega_f$

$\left(\frac{1}{2}m_cr_c^2\right)\left(7.00\frac{rad}{s}\right) = \left(\frac{1}{2}m_cr_c^2 + m_pr_p^2\right)\omega_f$

$\left(\frac{1}{2}(10kg)(1m)^2\right)\left(7\frac{rad}{s}\right) = \left(\frac{1}{2}(10kg)(1m)^2 + (0.250kg)(0.9m)^2\right)(\omega_f)$

$\omega_f = \frac{35\frac{kgm^2}{s}}{5.20kgm^2} = \boxed{6.73\frac{rad}{s}}$

8.



$I_c = mr^2$

$r_M = 2.00m$

$I_M = 275kgm^2$

$m_c = 25kg$

$r_{c_1} = 1.00m$

$\omega_i = \frac{14\,\cancel{rev}}{\cancel{min}}\cdot\frac{2\pi\,rad}{1\,\cancel{rev}}\cdot\frac{1\,\cancel{min}}{60s} = 1.47\frac{rad}{s}$

ω_f?

$r_{c_2} = 2.00m$

$L_i = L_f$

$I_i\omega_i = I_f\omega_f$

$(275kgm^2 + m_cr_{c_1}^2)\left(1.47\frac{rad}{s}\right) = (275kgm^2 + m_cr_{c_2}^2)(\omega_f)$

$(275kgm^2 + (25kg)(1.00m)^2)\left(1.47\frac{rad}{s}\right) = (275kgm^2 + 25kg(2.00m)^2)(\omega_f)$

$441\frac{kgm^2}{s} = 375kgm^2\,\omega_f$

$\boxed{\omega_f = 1.18\frac{rad}{s}}$

9. $m_w = 60kg$

$I_t = 500kgm^2$

$r_t = 2.00m$

$\omega_i = 0\frac{rad}{s}$

$V_w = 1.50\frac{m}{s}$

$\omega_t?$

$V = \omega r$

$I_w = m_w r_w^2$

$L_i = L_f$

$\cancel{I_i \omega_i} = I_f \omega_f$

$0 = I_w \omega_w - I_t \omega_t$

$I_t \omega_t = I_w \omega_w$

$$\omega_t = \frac{(m_w r_w^2)\left(\frac{V_w}{r_w}\right)}{500kgm^2}$$

$$\omega_t = \frac{(60kg)(2.00m)^2\left(\frac{1.50m/s}{2.00m}\right)}{500kgm^2}$$

$$\boxed{\omega_t = 0.36\frac{rad}{s}}$$