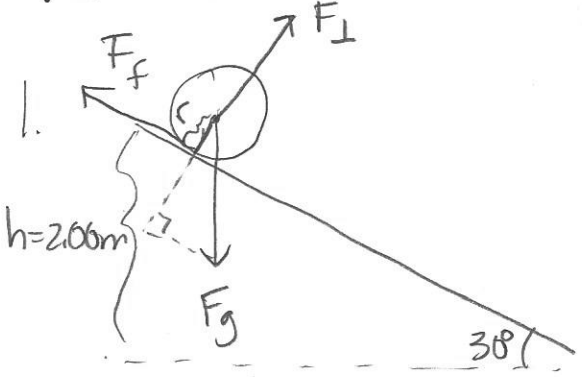
Worksheet 5 — AP Rotational Motion.

1.



$\Sigma \tau = F_f r = I\alpha$

$F_f R = \frac{2}{5}MR^2\left(\frac{a}{R}\right)$

$F_f = \frac{2}{5}Ma$

$\Sigma F = F_{gx} - F_f = ma$

$Mg\sin 30° - \frac{2}{5}Ma = Ma$

$g\sin 30° = a + \frac{2}{5}a$

$\frac{g\sin 30°}{\frac{7}{5}} = \frac{\frac{7}{5}a}{\frac{7}{5}}$

$a = 3.57 m/s^2$

$m = M$

$r = R$

$h = 2.00m$

$V?$

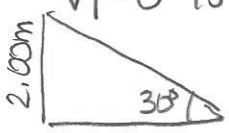
$I = \frac{2}{5}MR^2$

$\alpha = \frac{a}{r}$

$a = 3.57 m/s^2$

$\Delta X = 4.00m$

$V_i = 0 m/s$



$\sin 30° = \frac{2.00m}{\Delta X}$

$\Delta X = 4.00m$

$V_f^2 = V_i^2 + 2a\Delta X$

$V_f = \sqrt{2(3.57 m/s^2)(4.00m)}$

$\boxed{V_f = 5.34 m/s}$

OR ENERGY

$E_g = E_{K\,linear} + E_{K\,rot}$

$I = \frac{2}{5}MR^2$

$\omega = \frac{V}{R}$

$Mg\Delta y = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

$Mg\Delta y = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{V}{R}\right)^2$

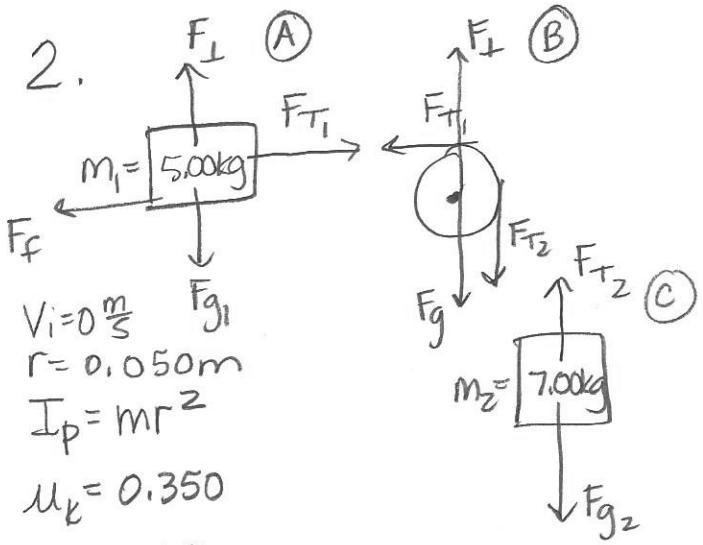
$g\Delta y = \frac{1}{2}V_f^2 + \frac{1}{5}V_f^2$

$g\Delta y = \frac{7}{10}V^2$

$V_f = \sqrt{\frac{10g\Delta y}{7}} = 5.35 m/s$

$\boxed{V_f = 5.35 \frac{m}{s}}$

2.



$V_i = 0 \frac{m}{s}$

$r = 0.050m$

$I_p = mr^2$

$\mu_k = 0.350$

$\Delta y = 2.00m$

$M_p = 2.00kg$

V_f ?

$F_\perp = F_{g_1} = m_1 g = (5.00kg)(10N/kg) = 50N$

$F_{g_2} = m_2 g = (7.00kg)(10N/kg) = 70N$

$\alpha = \frac{a}{r}$

(A) $\Sigma F_y = F_\perp - F_{g_1} = 0 \quad \Sigma F_x = F_{T_1} - F_f = m_1 a$

$F_\perp = F_{g_1}$ $\quad F_{T_1} - F_f = m_1 a$

$F_\perp = 50N$ $\quad F_{T_1} - \mu F_\perp = m_1 a$

$F_{T_1} - (0.35)(50N) = (5kg)a$

$F_{T_1} = (5kg)a + 17.5N$

(B) $\Sigma \tau = \tau_c - \tau_{cc} = F_{T_2} r - F_{T_1} r = I\alpha$

$F_{T_2}(0.050m) - F_{T_1}(0.050m) = mr^2\left(\frac{a}{r}\right)$

$(0.050m)(F_{T_2} - F_{T_1}) = (2.00kg)(0.050m)^2\left(\frac{a}{0.050m}\right)$

$F_{T_2} - F_{T_1} = (2kg)a$

(C) $\Sigma F = F_{g_2} - F_{T_2} = m_2 a$

$F_{g_2} - F_{T_2} = m_2 a$

$F_{T_2} = F_{g_2} - m_2 a$

$F_{T_2} = m_2 g - m_2 a$

$F_{T_2} = 70N - (7.00kg)a$

Put (A) & (C) into (B)

$F_{T_2} - F_{T_1} = (2kg)a$

$70N - (7.00kg)a - ((5kg)a + 17.5N) = (2kg)a$

$70N - (7kg)a - (5kg)a - 17.5N = (2kg)a$

$52.5N - (12kg)a = (2kg)a$

$52.5N = (14kg)a$

$a = 3.75 m/s^2$

$V_f^2 = V_i^2 + 2a\Delta y$

$V_f = \sqrt{2(3.75 m/s^2)(2.00m)}$

$\boxed{V_f = 3.87 \frac{m}{s}}$

OR ENERGY

$E_{g_2} = E_{k_1} + E_{k_2} + E_{k\,rot} + E_{th}$

$m_2 g\Delta y = \frac{1}{2} m_1 V_f^2 + \frac{1}{2} m_2 V_f^2 + \frac{1}{2} I\omega^2 + \mu m_1 g\Delta x$

$I = mr^2$

$\omega = \frac{v}{r}$

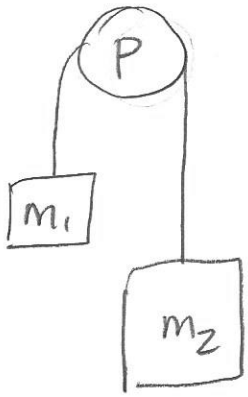
$m_2 g\Delta y = \frac{1}{2} m_1 V_f^2 + \frac{1}{2} m_2 V_f^2 + \frac{1}{2} m_p r^2 \left(\frac{V_f^2}{r^2}\right) + \mu m_1 g\Delta x$

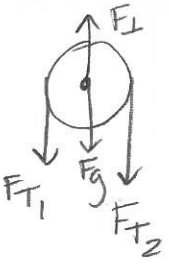
$70N(2.00m) = 2.50kg\,V_f^2 + 3.50kg\,V_f^2 + V_f^2 + (0.35)50N(2m)$

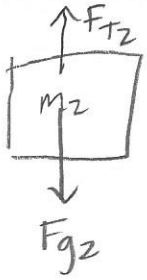
$140Nm = 7.0kg\,V_f^2 + 35Nm$

$V_f = \sqrt{\frac{105\,Nm}{7\,kg}} = \boxed{3.87 \frac{m}{s^2}}$

3.







$\Sigma F = F_{g_2} - F_{T_2} = m_2 a$

$F_{T_2} = F_{g_2} - m_2 a$

$F_{T_2} = 90N - (9kg)a$

$\Sigma F = F_{T_1} - F_{g_1} = m_1 a$

$\Sigma \tau = F_{T_2} r - F_{T_1} r = \frac{1}{2} M r^2 \left(\frac{a}{r}\right)$

$F_{T_2} - F_{T_1} = \frac{1}{2} M a$

$m_1 = 2.00kg$

$m_2 = 9.00kg$

$M_p = 3.00kg$

$r = 0.050m$

$v_i = 0 \frac{m}{s}$

$v_f = 2.50 \frac{m}{s}$

$\Delta y = 1.00m$

$E_{th} = ? = W_f = \tau\theta$

$F_{T_1} = F_{g_1} + m_1 a$

$F_{T_1} = 20N + (2kg)a$

$90N - (9kg)a - 20N - (2kg)a = \frac{1}{2}(3.00kg)a$

$70N - (11kg)a = (1.50kg)a$

$70N = (12.5kg)a$

$a = 5.6 m/s^2$

$E_{g_2} = E_{g_1} + E_{k_1} + E_{k_2} + E_{k\,rot} + E_{th}$

$m_2 g \Delta y = m_1 g \Delta y + \frac{1}{2} m_1 v_f^2 + \frac{1}{2} m_2 v_f^2 + \frac{1}{2} I \omega_f^2 + E_{th}$

$90N(1m) = (20N)(1m) + (1kg)v_f^2 + (4.50kg)v_f^2 + \frac{1}{2}\left(\frac{1}{2} m r^2 \left(\frac{v^2}{r^2}\right)\right) + E_{th}$

$90Nm = 20Nm + (1kg)v_f^2 + 4.50kg(v_f^2) + \frac{1}{4}(3.00kg)v_f^2 + E_{th}$

$90Nm = 20Nm + 6.25Nm + 28.13Nm + 4.69Nm + E_{th}$

$31Nm = E_{th}$

$\boxed{31J = E_{th}}$

4. $I_i = 2.25\ kgm^2$

$\omega_i = \frac{1\ rev}{1.26\ s} \left| \frac{2\pi\ rad}{1\ rev} \right| = 5\ rad/s$

$I_f = 1.80\ kgm^2$

ω_f ?

$I_i\omega_i = I_f\omega_f$

$\omega_f = \frac{I_i\omega_i}{I_f} = \frac{(2.25\ kgm^2)(5\ rad/s)}{1.80\ kgm^2}$

$\omega_f = 6.25\ rad/s$

$W = \Delta E_k = \frac{1}{2} I_f \omega_f^2 - \frac{1}{2} I_i \omega_i^2 = \frac{1}{2}(1.80\ kgm^2)(6.25\ rad/s)^2$

$- \frac{1}{2}(2.25\ kgm^2)(5\ rad/s)^2$

$= 35.2\ J - 28.1\ J = \boxed{7.1\ J}$