Calculating pH

How is pH related to the concentration of hydronium ions?

Why?

In biology and other science courses pH is introduced as a way to quantify the acidity or basicity of a solution. This property can be measured using a pH probe or with an indicator paper strip that changes color at different pH values. But, what is actually being measured? We know that a pH of 7 is neutral, below 7 is acid, and above 7 is base, but why? What in the solution is the paper strip or probe actually reacting with?

Model 1 – Ion Concentrations for Acids and Bases

<table>
<thead>
<tr>
<th>Beaker</th>
<th>Solution</th>
<th>Acidic, Basic or Neutral?</th>
<th>$[\text{H}_3\text{O}^+]$</th>
<th>$[\text{OH}^-]$</th>
<th>$[\text{H}_3\text{O}^+] \times [\text{OH}^-]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.10 M HCl(aq)</td>
<td>Acidic</td>
<td>$1.0 \times 10^{-1}$ M</td>
<td>$1.0 \times 10^{-13}$ M</td>
<td>$1 \times 10^{-14}$</td>
</tr>
<tr>
<td>2</td>
<td>0.0010 M HCl(aq)</td>
<td>Acidic</td>
<td>$1.0 \times 10^{-3}$ M</td>
<td>$1.0 \times 10^{-11}$ M</td>
<td>$1 \times 10^{-14}$</td>
</tr>
<tr>
<td>3</td>
<td>0.000010 M HCl(aq)</td>
<td>Acidic</td>
<td>$1.0 \times 10^{-5}$ M</td>
<td>$1.0 \times 10^{-9}$ M</td>
<td>$1 \times 10^{-14}$</td>
</tr>
<tr>
<td>4</td>
<td>0.00000010 M HCl(aq)</td>
<td>Acidic</td>
<td>$1.0 \times 10^{-6}$ M</td>
<td>$1.0 \times 10^{-8}$ M</td>
<td>$1 \times 10^{-14}$</td>
</tr>
<tr>
<td>5</td>
<td>0.00000010 M HCl(aq)</td>
<td>Neutral</td>
<td>$1.0 \times 10^{-7}$ M</td>
<td>$1.0 \times 10^{-7}$ M</td>
<td>$1 \times 10^{-14}$</td>
</tr>
<tr>
<td>6</td>
<td>0.00000010 M NaOH(aq)</td>
<td>Neutral</td>
<td>$1.0 \times 10^{-7}$ M</td>
<td>$1.0 \times 10^{-7}$ M</td>
<td>$1 \times 10^{-14}$</td>
</tr>
<tr>
<td>7</td>
<td>0.00000010 M NaOH(aq)</td>
<td>Basic</td>
<td>$1.0 \times 10^{-8}$ M</td>
<td>$1.0 \times 10^{-6}$ M</td>
<td>$1 \times 10^{-14}$</td>
</tr>
<tr>
<td>8</td>
<td>0.00010 M NaOH(aq)</td>
<td>Basic</td>
<td>$1.0 \times 10^{-10}$ M</td>
<td>$1.0 \times 10^{-4}$ M</td>
<td>$1 \times 10^{-14}$</td>
</tr>
<tr>
<td>9</td>
<td>0.010 M NaOH(aq)</td>
<td>Basic</td>
<td>$1.0 \times 10^{-12}$ M</td>
<td>$1.0 \times 10^{-2}$ M</td>
<td>$1 \times 10^{-14}$</td>
</tr>
<tr>
<td>10</td>
<td>0.10 M NaOH(aq)</td>
<td>Basic</td>
<td>$1.0 \times 10^{-13}$ M</td>
<td>$1.0 \times 10^{-1}$ M</td>
<td>$1 \times 10^{-14}$</td>
</tr>
</tbody>
</table>

1. What does the symbol $[\text{H}_3\text{O}^+]$ in Model 1 indicate?
   *The molar concentration of hydronium ion.*

2. In Beaker 2, which ion has a higher concentration, hydronium ion or hydroxide ion?
   *Hydronium ion.*

3. Describe how the concentration of hydronium ion was calculated for Beaker 3 in Model 1 from the concentration of the acid.
   *Since HCl is a strong acid, all of the molecules ionize in water. Therefore, the $\text{H}_3\text{O}^+$ concentration is equal to the initial acid concentration in the solution.*

4. Describe how the concentration of hydroxide ion was calculated for Beaker 8 in Model 1 from the concentration of the base.
   *Since NaOH is a strong base, it completely dissociates in water. Therefore, the $\text{OH}^-$ concentration is equal to the initial base concentration in the solution.*
5. Which ion, hydronium or hydroxide, has a higher concentration in an acidic solution?

*Hydronium ions are present in a higher concentration than hydroxide ions in an acidic solution.*

6. Which ion, hydronium or hydroxide, is more concentrated in a neutral solution?

*The concentrations of hydronium ions and hydroxide ions are equal in a neutral solution.*

7. Which statement is true for basic solutions?

   a. The hydroxide ion concentration must be less than $1.0 \times 10^{-7}$ M.
   
   b. The hydroxide ion concentration must be greater than $1.0 \times 10^{-7}$ M.
   
   c. The hydroxide ion concentration must be more than or equal to $1.0 \times 10^{-7}$ M.

8. A student makes the following statement on an exam: “Acidic solutions contain hydronium ions, while basic solutions contain hydroxide ions.” Is the student’s statement correct based on the information in Model 1? Explain.

*No—all of the solutions contain both hydronium and hydroxide ions. The acidic solutions have more hydronium ions and the basic solutions have more hydroxide ions.*

9. Calculate the quantity $[\text{H}_3\text{O}^+] \times [\text{OH}^-]$ for each of the 10 beakers in Model 1. Divide the work among the members in your group.

*See Model 1 above.*

10. If you know the hydronium ion concentration, $[\text{H}_3\text{O}^+]$, of a solution, how could you determine the hydroxide ion concentration, $[\text{OH}^-]$?

   \[
   [\text{H}_3\text{O}^+] \times [\text{OH}^-] = 1.0 \times 10^{-14}
   \]

   Enter the known value of $[\text{H}_3\text{O}^+]$ in the equation and solve for $[\text{OH}^-]$.

11. A solution has a hydronium ion concentration of $1.0 \times 10^{-3}$ M.

   a. What is the hydronium ion concentration in the solution? (Show your work.)

   \[
   [\text{H}_3\text{O}^+] \times [\text{OH}^-] = 1.0 \times 10^{-14}
   
   [\text{H}_3\text{O}^+] \times (1.0 \times 10^{-3} \text{ M}) = 1.0 \times 10^{-14}
   
   [\text{H}_3\text{O}^+] = 1.0 \times 10^{-11} \text{ M}
   \]

   b. Is the solution acidic, neutral or basic? How do you know?

   *The solution is basic because the $[\text{OH}^-]$ is greater than $1.0 \times 10^{-7}$ M. Alternatively, the solution can be classified based on $[\text{H}_3\text{O}^+]$, which is less than $1.0 \times 10^{-7}$ M.*

12. A solution has a hydronium ion concentration of $4.79 \times 10^{-3}$ M.

   a. What is the hydronium ion concentration in the solution? (Show your work.)

   \[
   1.0 \times 10^{-14} = [\text{H}_3\text{O}^+] \times [\text{OH}^-]
   
   1.0 \times 10^{-14} = [\text{H}_3\text{O}^+] \times 4.79 \times 10^{-3} \text{ M}
   
   [\text{H}_3\text{O}^+] = 2.1 \times 10^{-12} \text{ M}
   \]

   b. Is the solution acidic, neutral or basic? How do you know?

   *The solution is basic because $[\text{OH}^-]$ is greater than $1.0 \times 10^{-7}$ M. Alternatively, $[\text{H}_3\text{O}^+]$ is less than $1.0 \times 10^{-7}$ M.*
Read This!

The value $1.0 \times 10^{-14}$ is the equilibrium constant for the autoionization of water ($K_w$).

$$H_2O + H_2O \rightleftharpoons H_3O^+(aq) + OH^-(aq) \quad K_w = 1.0 \times 10^{-14}$$

This equilibrium occurs in all aqueous solutions (acidic, basic, and neutral). The results of this equilibrium are as follows:

1. All aqueous solutions have some detectable concentration of both hydronium and hydroxide ions.
2. The product of these ion concentrations is always $K_w$.

$$K_w = [H_3O^+] \times [OH^-] = 1.0 \times 10^{-14}$$

Model 2 – A Crash Course in Logarithms

<table>
<thead>
<tr>
<th>log 1</th>
<th>log 10</th>
<th>log 100</th>
<th>log 1000</th>
<th>log (1.0×10^6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>log 0.1</td>
<td>log 0.01</td>
<td>log 0.001</td>
<td>log (1.0×10^-4)</td>
<td>log (1.0×10^-8)</td>
</tr>
<tr>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td>-8</td>
</tr>
</tbody>
</table>

13. Using the examples in Model 2, explain how logarithms are calculated in terms of “factors of ten.”

*A logarithm is the exponent on the “×10” part of a number in scientific notation. If a number is in the thousands, it has three “factors of ten” so the log will be 3.*

14. What would be the logarithm of one million? (Do NOT use your calculator.)

$$\log (1.0 \times 10^6) = 6$$

15. Take out your scientific calculator.

a. Enter at least three of the examples shown in Model 2 into your calculator to verify that you know how to find the logarithm of a number.

b. Use your calculator to find the logarithm of 250.

$$2.39$$

c. The number 250 is between 100 and 1000. Explain why your calculator gave you an answer between 2 and 3 for the log of 250. *Hint: Think about “factors of ten.”*

*The logarithm of 250 must be somewhere between log 100 = 2 and log 1000 = 3.*

16. First estimate the answer for each of the following. Then, find the answer using your calculator to check your estimate.

a. $7800$  b. $0.045$  c. $3.4 \times 10^9$  d. $7.2 \times 10^{-4}$

$$3.89 \quad -1.35 \quad 9.53 \quad -3.14$$

Calculating pH
Model 3 – Logarithms and pH

<table>
<thead>
<tr>
<th>Solution</th>
<th>1 [H$_3$O$^+$] (Decimal notation)</th>
<th>2 [H$_3$O$^+$] (Scientific notation)</th>
<th>3 log [H$_3$O$^+$]</th>
<th>4 pH</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.010 M</td>
<td>1.0×10$^{-2}$ M</td>
<td>log (1.0×10$^{-2}$) = −2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>B</td>
<td>0.0055 M</td>
<td>5.5×10$^{-3}$ M</td>
<td>log (5.5×10$^{-3}$)</td>
<td>2.3</td>
</tr>
<tr>
<td>C</td>
<td>0.0010 M</td>
<td>1.0×10$^{-3}$ M</td>
<td>log (1.0×10$^{-3}$) = −3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>D</td>
<td>0.00010 M</td>
<td>1.0×10$^{-4}$ M</td>
<td>log (1.0×10$^{-4}$) = −4.0</td>
<td>4.0</td>
</tr>
<tr>
<td>E</td>
<td>0.000027 M</td>
<td>2.7×10$^{-5}$ M</td>
<td>log (2.7×10$^{-5}$)</td>
<td>4.6</td>
</tr>
</tbody>
</table>

17. Columns 1 and 2 in Model 3 both give the molar concentration of hydronium ion in solution.
   a. What is the difference in the way the first two columns express this data?
   *Decimal notation vs. scientific notation.*
   b. Fill in the missing values in columns 1 and 2 of Model 3.
   See Model 3 above.

18. Estimate the missing logarithms for solutions B and E in Model 3. Then verify the answers using a calculator.
   See Model 3 above.

19. Using the examples given in Model 3, write a sentence or a mathematical equation that describes how to calculate pH from the hydronium ion concentration of a solution.
   To find pH, find the log of the hydronium ion concentration and reverse the sign. pH = −log [H$_3$O$^+$].

20. Fill in the missing pH values in column 4 of Model 3.
   See Model 3 above.

21. Calculate the pH of a solution that has a hydronium ion concentration of:
   a. 1×10$^{-4}$ M
   pH = 8
   b. 0.007 M
   pH = 2.2

22. Discuss in your group how you would find the hydronium ion concentration in a solution if you were given the pH. Check your procedure using several examples from Model 3.
   *The hydronium ion concentration is determined by calculating the value of 10 raised to the power of the negative pH.*
   [H$_3$O$^+$] = 10$^{-pH}$.

23. Calculate the hydronium ion concentration in solutions with a pH of:
   a. 6.0
   $[H_3O^+] = 1 \times 10^{-6} M$
   b. 5.43
   $[H_3O^+] = 10^{-5.43} = 3.7 \times 10^{-6} M$

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24. Why does neutral water have a pH of 7?

When the hydronium and hydroxide ions are equal, they both have a concentration of $1.0 \times 10^{-7}$ M. The negative logarithm of that concentration is 7.

25. Which solution has a greater hydronium ion concentration, one that has a pH of 4 or one that has a pH of 8? Explain.

$pH$ of 4 has more hydronium ions. The pH number is a negative exponent, so $1 \times 10^{-4}$ M is greater than $1 \times 10^{-8}$ M.

26. A student makes the following statement on an exam:

"A solution with pH = 1 is twice as concentrated in hydronium ions as a solution with pH = 2."

Explain why this statement is not correct, and write a sentence that describes the correct relationship.

$pH = 1$ means $[H_3O^+] = 0.1$ M

$pH = 2$ means $[H_3O^+] = 0.01$ M

The pH = 1 solution has 10 times more hydronium ions than the pH = 2 solution.
Extension Questions

Model 4 – pH and pOH

<table>
<thead>
<tr>
<th>Solution</th>
<th>[H$_2$O$^+$]</th>
<th>[OH$^-$]</th>
<th>pH</th>
<th>pOH</th>
<th>pH + pOH</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1x10^{-3} M</td>
<td>1x10^{-11} M</td>
<td>3.0</td>
<td>11.0</td>
<td>14</td>
</tr>
<tr>
<td>B</td>
<td>1x10^{-9} M</td>
<td>1x10^{-5} M</td>
<td>9.0</td>
<td>5.0</td>
<td>14</td>
</tr>
<tr>
<td>C</td>
<td>5.2x10^{-3} M</td>
<td>1.9x10^{-12} M</td>
<td>2.28</td>
<td>11.72</td>
<td>14</td>
</tr>
<tr>
<td>D</td>
<td>4.79x10^{-11} M</td>
<td>2.09x10^{-4} M</td>
<td>10.32</td>
<td>3.68</td>
<td>14</td>
</tr>
<tr>
<td>E</td>
<td>5.25x10^{-10} M</td>
<td>1.91x10^{-5} M</td>
<td>9.28</td>
<td>4.72</td>
<td>14</td>
</tr>
<tr>
<td>F</td>
<td>3.31x10^{-12} M</td>
<td>3.02x10^{-3} M</td>
<td>11.48</td>
<td>2.52</td>
<td>14</td>
</tr>
</tbody>
</table>

27. Look at the examples in Model 4. If you know the concentration of hydroxide ion, [OH$^-$], in a solution, how can you determine the pOH?

\[
\text{pH} = -\log [\text{OH}^-]
\]

28. Consider the data in Model 4.

a. Calculate pH + pOH for solutions A, B and C.

See Model 4 above.

b. How could you determine the pH of a solution if you know the pOH?

Since pH + pOH = 14, substitute the known value of pOH and solve for pH.

29. Fill in all of the missing values in Model 4.

30. Calculate the [OH$^-$] and pOH of a solution that has a [H$_3$O$^+$] of 1x10^{-4} M.

\[
[H_3O^+] = 1 \times 10^{-4} M \quad \text{pH} = 4
\]

\[
[OH^-] = 1 \times 10^{-10} M \quad \text{pOH} = 10
\]